TME225 Mechanics of fluids, 7th November 2012

Assignment 2: Turbulent flow

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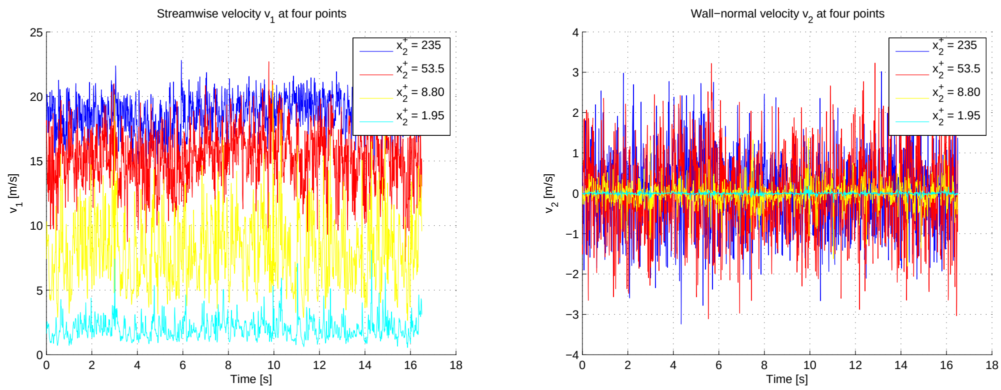


Figure 1: *The streamwise velocity v_1 (left) and the wall-normal velocity v_2 (right) for the four different points.*

Table 1: *Difference between true mean and mean for fewer samples at node 1.*

Samples	Difference (m/s)	Difference (%)
100	0.462	22.8
500	0.203	10.0
1000	0.054	2.7
2500	0.018	0.9

1 Time history

The time history for the velocity in four different points in a fully developed turbulent channel flow can be seen in figure 1. As expected, the velocities are oscillating considerably, which is one of the characteristic features of turbulence. Furhermore, the mean of the streamwise velocity increases when x_2 increases, which of course follows from the fact that $\frac{\partial \bar{v}_1}{\partial x_2} > 0$ in the lower half of the channel.

If we consider the wall-normal velocity v_2, we find that its mean seems to be zero for all four points, which should be the case since the theory states that $\bar{v}_2 = 0$. However, the fluctuating part v_2' oscillates heavier when x_2 increases. This is also expected, since larger fluctuations are possible further out from the wall.

2 Time averaging

The time average of the streamwise velocity in the four nodes can be seen in figure 2 on the next page.

Table 2: *Maximum and minimum of v_1 at the four nodes.*

x_2^+	v_1^{min} (m/s)	v_1^{mean} (m/s)	v_1^{max} (m/s)
1.95	0.55	2.03	8.09
8.80	2.65	7.96	20.88
53.5	7.23	15.14	22.70
235	14.28	18.61	22.80

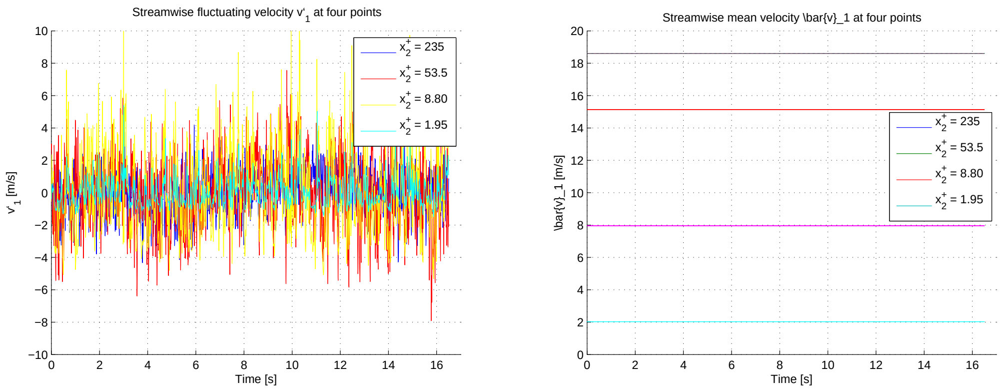


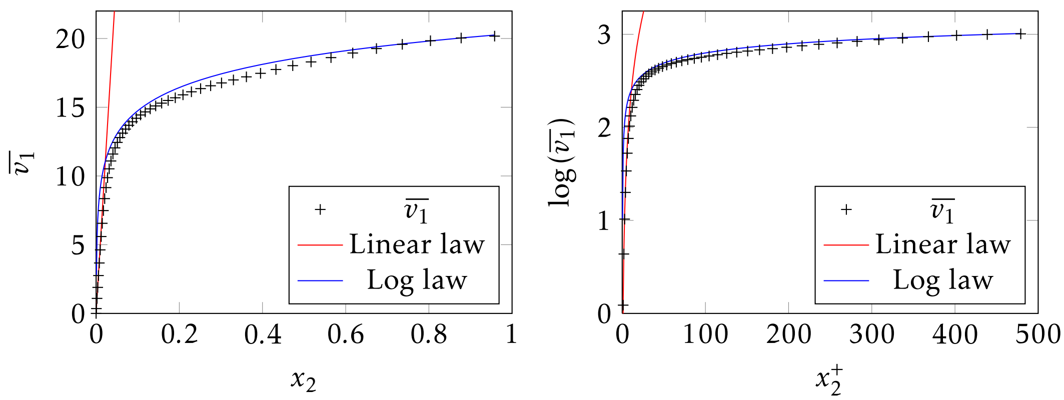
Figure 2: *The streamwise fluctuating velocity v_1' (left) and mean velocity $\bar{v}_2$ (right) for the four different points.*

Table 1 on the preceding page shows the mean calculated from different number of samples. We can see that a large number of samples is required to achieve accuracy. The error doesn't drop below 1 % until half the samples are used in the calculation.

The maximum and minimum value of the streamwise velocity at the four nodes is shown in table 2. As expected, the streamwise velocity increases as we move away from the wall. This is true for both the mean value and the extreme values.

3 Mean flow

Figure 3 on the following page shows the mean velocity in the x_1 direction for the lower half of the flow. It is fairly clear from the graph that the velocity profile follows the linear law until $x_2^+ \approx 5$, which roughly corresponds to what is refered to as the viscous region in Davidson (2012, p. 53). The log law is followed fairly well from $x_2^+ \approx 50$ and very well from $x_2^+ \approx 250$ and onwards. This also corresponds to the log-law region discussed in Davidson (2012, p. 53).



(a) Linear comparison plot

(b) Log-linear comparison plot

Figure 3: *The mean velocity $\overline{v_1}$ compared to the linear law and the log law.*

The bulk velocity can be computed as

$$V_{1,b} = \frac{1}{2h} \int_0^{2h} \overline{v_1} \, \mathrm{d}x_2 = 17.55\,\mathrm{m/s},$$

and the centerline velocity is trivially fetched from the data and is equal to $V_{1,c} = 20.17\,\mathrm{m/s}$. From these we can calculate the corresponding Reynolds numbers. Basing the Reynolds number on half the channel width like the Reynolds number given in the assignment, we can calculate these new Reynolds numbers as $\mathrm{Re}_i = {}^{V_{1,i}h}/_{\nu}$, which yields the values $\mathrm{Re}_b = 8.77 \times 10^3$ and $\mathrm{Re}_c = 10.01 \times 10^4$.

4 The time-averaged momentum equation

As seen in figure 4 on the next page, the terms of the streamwise momentum equation,

$$0 = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_1} + \nu \frac{\partial^2 \overline{v_1}}{\partial x_2^2} - \frac{\partial \overline{v_1' v_2'}}{\partial x_2},$$

roughly cancel everywhere except at the walls, where the Reynolds stress term is very large, which is confirmed by figure 6 on page 6 as well. The viscous term (the second term) remains negative except at the very edges of the flow, and from Davidson (2012, p. 52) one should expect it to be cancelled (as in, their sum will be −1) by the Reynolds stress term. This is in fact roughly the case if we stay away from the walls. The viscous term is largest at the walls. The Reynolds stress term is positive near the walls and negative in the center of the flow.

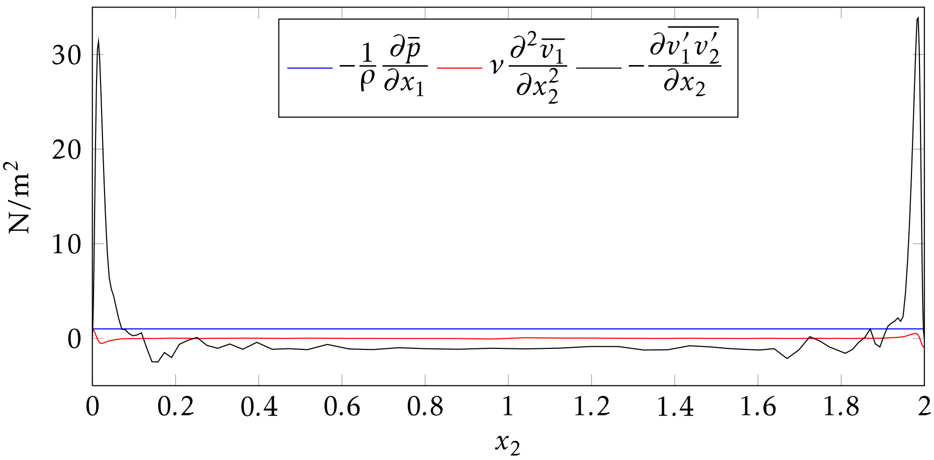


Figure 4: *The three terms of the streamwise momentum equation.*

5 Wall shear stress

The wall shear stress at the lower wall is given by

$$\tau_0 = \left.\frac{\partial v_1}{\partial x_2}\right|_{x_2=0},$$

where x_2 of course is the distance from the lower wall.

Figure 5 on the next page shows both τ_0 and τ_L plotted over time. Clearly, these are not equal, which of course is a result of turbulence.

6 Resolved stresses

Figure 6 on the following page clearly shows that among the stresses, only $\overline{v_2' v_3'}$ and $\overline{v_3' v_1'}$ are near zero. It is also apparent that the normal stresses, particularly $\overline{v_1' v_1'}$, are much larger than the shear stresses by a factor of almost 10.

7 Fluctuating wall shear stress

The root-mean-square of the wall shear stress is easily calculated using standard MATLAB functions, obtaining the values $\tau_{\mathrm{rms},0} = 374\,\mathrm{mPa}$ and $\tau_{\mathrm{rms,L}} = 400\,\mathrm{mPa}$. Looking at figure 5 on the next page, this is reasonable, since the root-mean-square as it is defined in the assignment is actually the standard deviation of the variable.

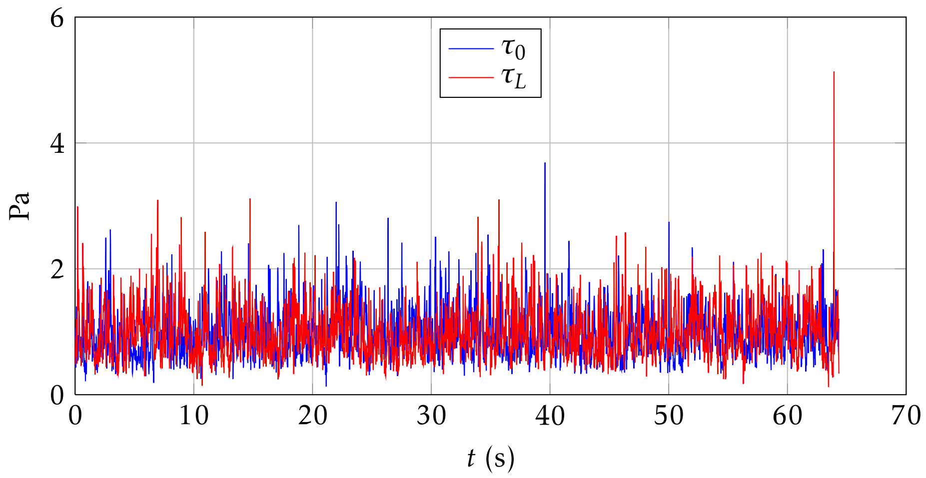
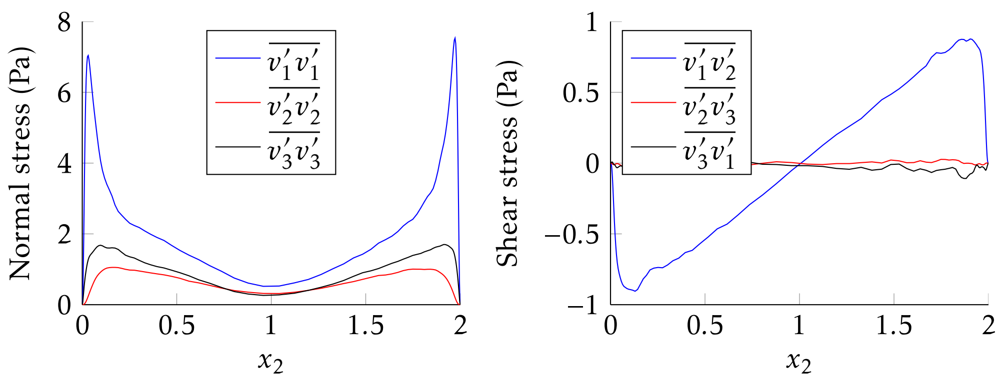


Figure 5: *The wall shear stress at the lower and upper wall over time.*



(a) Normal stresses

(b) Shear stresses

Figure 6: *The normal and shear stresses* $\overline{v_i'v_j'}$.

8 Production terms

The production terms related to the normal and shear stresses discussed earlier, calculated as $P_{ij} = -\overline{v_i'v_k'}\,{}^{\partial \bar{v}_j}/_{\partial x_k} - \overline{v_j'v_k'}\,{}^{\partial \bar{v}_i}/_{\partial x_k}$, can be seen in figure 7 on the following page. For the normal stresses, the production terms are fairly close to zero with the exception of P_{11} which is very large. The production terms for the shear stresses are small in comparison, even P_{12} whose corresponding stress is non-zero.

Only one of the production terms, P_{12}, changes sign at the centerline. The corresponding shear stress, $\overline{v_1'v_2'}$ does the same thing as figure 6 on the previous page clearly shows.

9 Pressure-strain terms

Since the equations solved in order to obtain our data doesn't include the term $\frac{\partial p}{\partial t}$, the pressure may vary in a non-physical way. This means that we probably will get better results if we compute the velocity-pressure gradient term instead of the pressure-strain term, since the difference between these two are negligible except very close to the wall. In figure 8 on the following page, the velocity-pressure gradient term can be seen for the three normal stresses, as well as for the cross-channel shear stress.

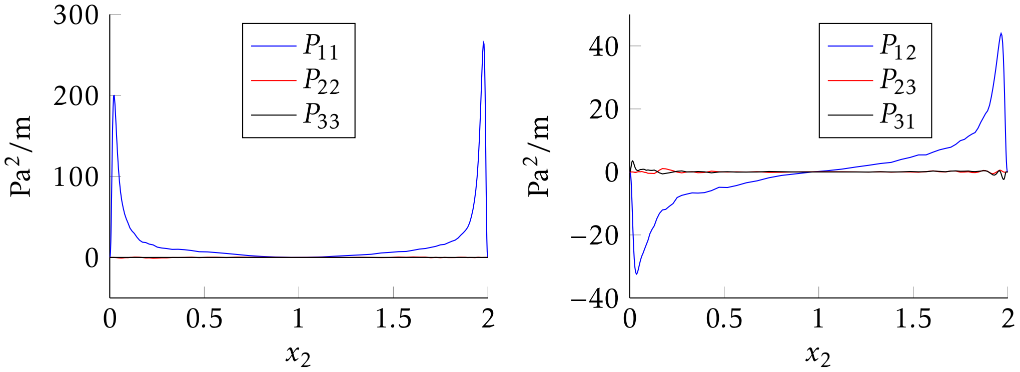
We see that the velocity-pressure gradient terms for the normal stresses are somewhat symmetric around the center of the channel, while the term for the shear stress is anti-symmetric. This is due to the "Robin Hood property" of the pressure strain term. Comparing figure 8 on the next page to figure 7 on the following page, we can see that P_{ij} and Π_{ij} have opposite signs.

The largest sink is clearly Π_{11} while the largest source is Π_{33}.

10 Dissipation

In figure 9 on page 9, the turbulent dissipation ε can be seen together with the mean flow dissipation $\varepsilon_{\text{mean}}$ and the production term P^k. We see that the mean flow dissipation is is very large at the wall and dominates for $x_2^2 \lesssim 20$, but decreases rapidly. For $x_2^2 \gtrsim 50$, almost all dissipation is due to the turbulent fluctuations.

By integrating ε and $\varepsilon_{\text{mean}}$, we find that the major part of the kinetic energy is transformed directly from K to ΔT.



(a) Normal production terms

(b) Shear production terms

Figure 7: *The production terms P_{ij}.*

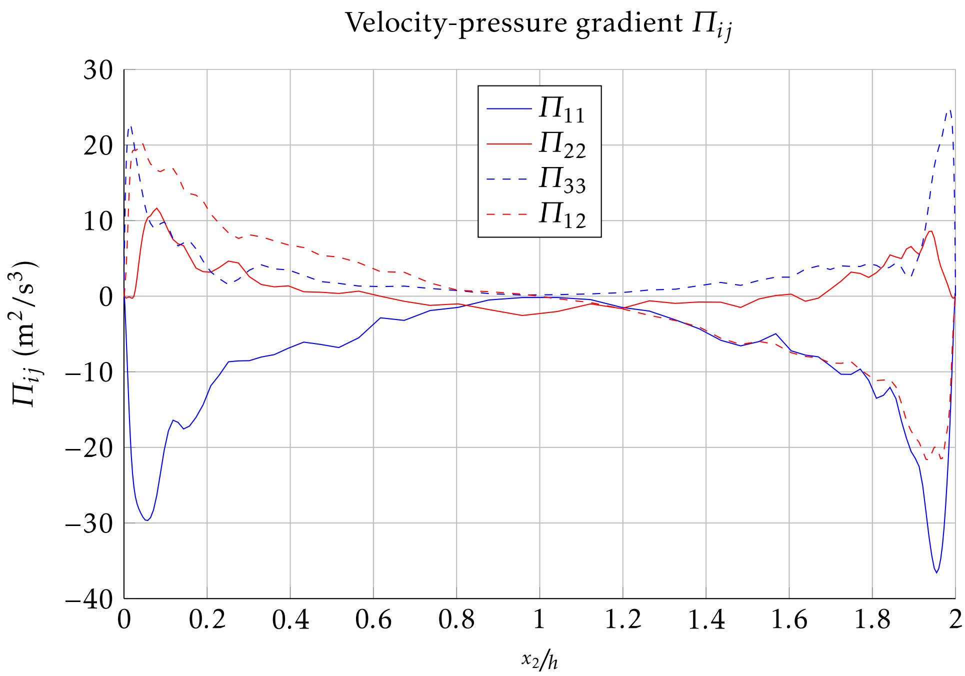


Figure 8: *The velocity-pressure gradient term for the three normal stresses and the cross-channel shear stress.*

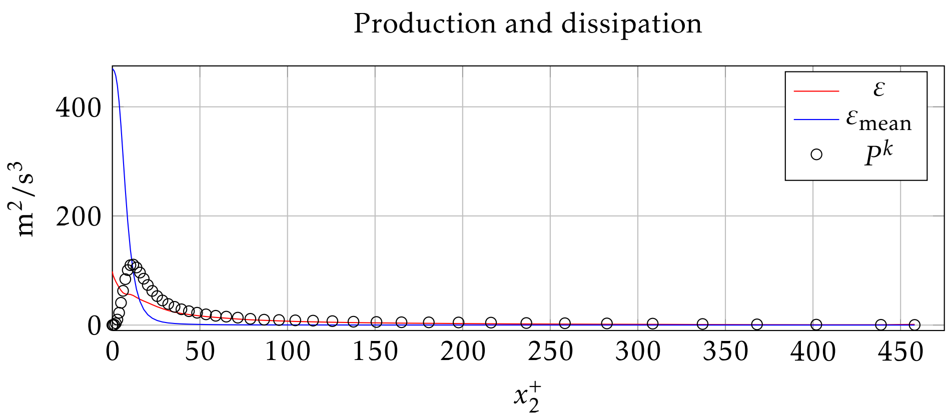


Figure 9: *The turbulent and mean flow dissipation together with the production term.*

11 The fun part – Autocorrelation

Since the autocorrelation actually is a convolution of $v_1'(t)$ with itself, we can use that $\widehat{(f \star g)}(t) = \hat{f}(\tau)\hat{g}(\tau)$ to compute it. To do this, we compute the Fourier transform of v_1', multiply it with its conjugate and perform the inverse transform. For details, please see the attached `MATLAB`-code.

The autocorrelation B_{11} for the four nodes from section 1 can be seen in figure 10 on the following page, where a maximum lag of 100 time steps has been used in the plot. We can see that the autocorrelation behaves as expected for all four nodes, that is, it starts at a value of 1 at no lag and falls towards zero.

From the autocorrelation, we can compute the integral time scale T_{int} for the flow as

$$T_{\text{int}} = \int_0^\infty B_{11}^{\text{norm}}(\hat{t})\,\mathrm{d}\hat{t}.$$

If we use a maximum time lag $\hat{t}$ of 100 time steps, we get the integral time scales in table 3.

Table 3: *Integral time scale T_{int} for the four nodes.*

x_2^+	T_{int} (s)
1.95	0.034
8.80	0.029
53.5	0.032
235	0.029

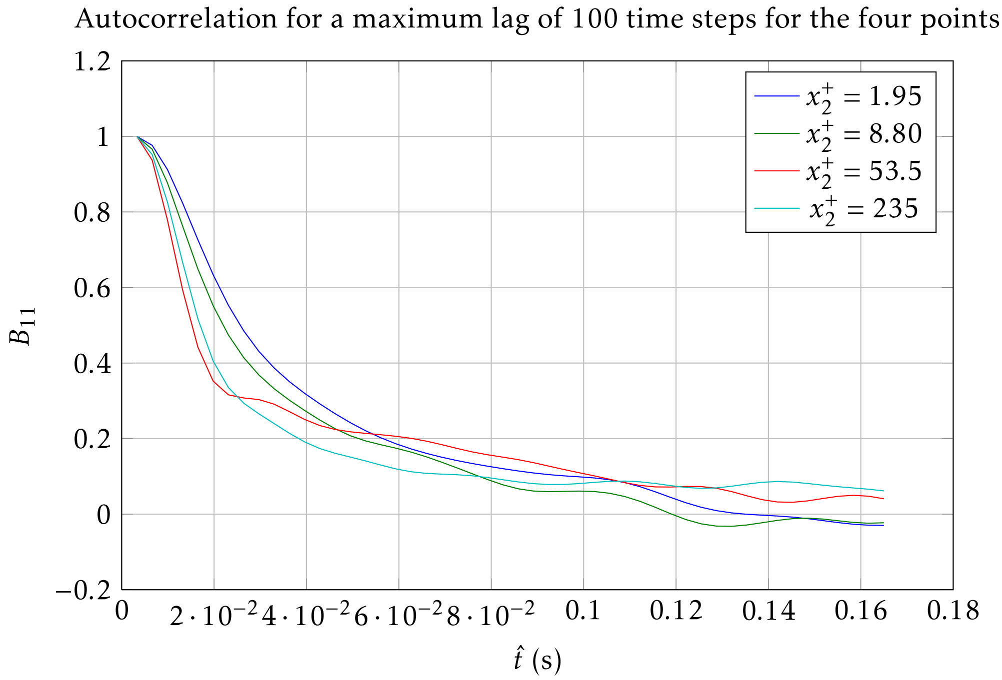


Figure 10: *The turbulent and mean flow dissipation together with the production term.*

References

Davidson, L. (2012) Fluid mechanics, turbulent flow and turbulence modeling. http://www.tfd.chalmers.se/~lada/MoF/lecturenotes.html.