TME225 Mechanics of fluids, 1st October 2012

Assignment 1: Laminar flow

Simon Sigurdhsson

1 Fully developed region

Testing for a small derivative tells us that the fully developed region begins at $x_1 = 0.4573\,\mathrm{m}$, while testing for 99% of the top speed instead indicates a fully developed region already at $x_1 = 0.2596\,\mathrm{m}$. The formula given in the lecture notes, $0.016\frac{V2h}{\nu}$, with $\nu = 1.478 \times 10^{-5}\,\mathrm{m^2/s}$, gave the obviously incorrect value $x_1 = 22.7712\,\mathrm{m}$.

In the fully developed region, we would (according to 3.2.2 in the lecture notes) expect $v_2 = 0$. In fact, at $x_2 = h/4$ inside the fully developed region (specifically at $x_1 = 0.560\,27\,\mathrm{m}$), we have $v_2 = 2.6867 \times 10^{-6}\,\mathrm{m/s}$ which is quite near 0.

2 Wall shear stress

Combining equations 1.5 and 2.3 directly yield $\tau_{ij} = 2\mu S_{ij} - 2/3\mu S_{kk}\delta_{ij}$, where $S_{kk} = 0$ due to incompressibility. This means that for the given n_j, we have $t_i = \tau_{ij} n_j = -\mu\left(\frac{\partial v_i}{\partial x_2} + \frac{\partial v_2}{\partial x_i}\right)$ and hence (since $\frac{\partial v_2}{\partial x_1} = 0$) the wall shear stress at the upper wall is $\tau_{\omega,U} = -\mu \left.\frac{\partial v_1}{\partial x_2}\right|_U$.

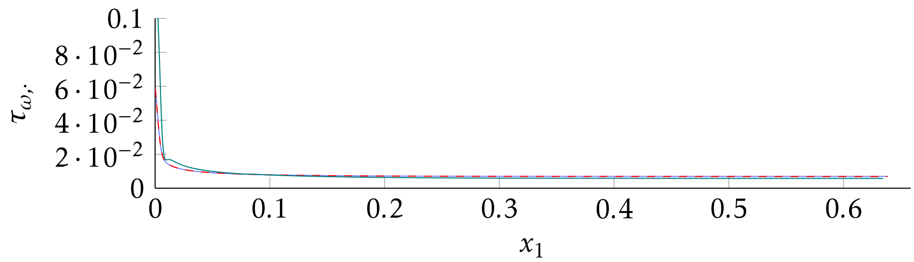
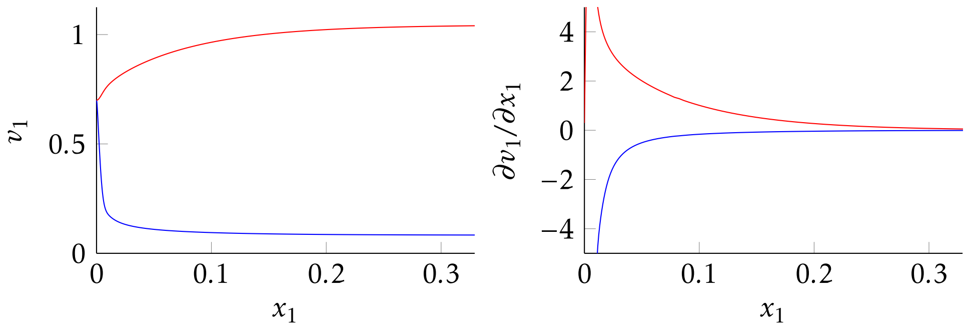


Figure 1: *Wall shear stress at lower (blue) and upper (red) wall.*



(a) Values of v_1

(b) Values of $\frac{\partial v_1}{\partial x_1}$

Figure 2: *Velocity of the fluid and its derivative near the wall (blue) and in the center (red)*

Figure 1 shows the shear stress at both walls, and it is obvious that the shear stress is high in the inlet, sharply decreasing to a very low level that is maintained in the fully developed flow. This is because the flow near (but not at) the walls has a high velocity at the inlet, before stabilizing once reaching the fully developed area.

3 Inlet region

Integrating the velocity w.r.t x_2 to get a function $\xi(x_1)$ should yield a constant function, since the flow is governed by the continuity equations. Calculating ξ using the given data reveals that it is, for all intents and purposes, constant.

4 Wall-normal velocity in the developing region

The velocity v_2 doesn't behave at all like v_1; in the center, starting at 0, it quickly rises to a quite small value of 0.025 m/s before rather quickly falling towards 0 again. It behaves similarly at the lower wall, with a lower magnitude, and at the upper wall with an opposite sign.

This could be explained as the fluid moving towards the middle of the stream to fill the gap created by the lower pressure from the faster-moving fluid at the center. Once this pressure difference is resolved, the flow is steady.

5 Vorticity

Since we have $\omega_3 = \frac{\partial v_1}{\partial x_2} - \frac{\partial v_2}{\partial x_1}$, with the latter term being 0 in the fully developed flow, there is no particular reason to think that the flow should be irrotational. In

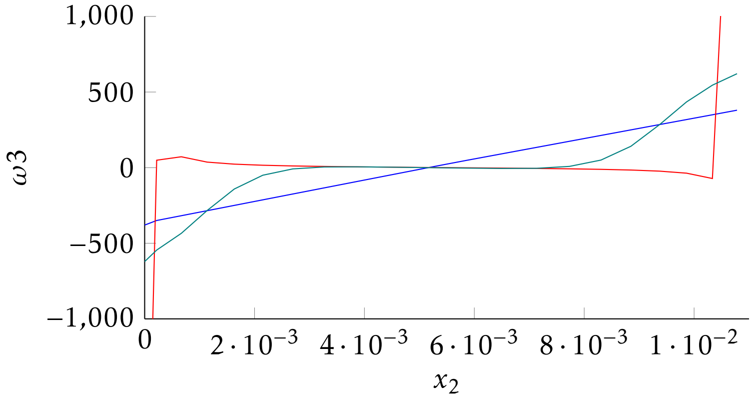


Figure 3: *Vorticity component ω_3 in the fully developed region (blue), the inlet (red) and the developing region (teal)*

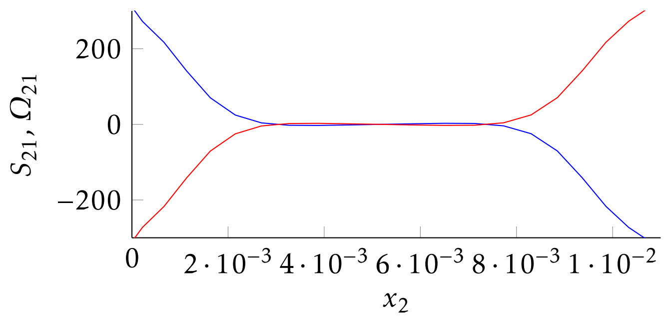


Figure 4: *The strain-rate tensor S_{ij} (blue) and the vorticity tensor Ω_{ij} (red) along x_2 in the developing area*

fact, as shown by figure 3, the flow is irrotational only in the center (where it also has the highest velocity). We can see that the whole width of the flow is very nearly irrotational in the inlet, but with time as the flow moves towards the fully developed region, the vorticity increases.

6 Deformation

The only two off-diagonal terms of the strain-rate tensor are S_{12} and S_{21}. Figure 4 shows one of these, along with the corresponding term of the vorticity tensor $\boldsymbol{\Omega}$. It is immediately obvious that the vorticity tensor is very similar to the vorticity in figure 3, which is not surprising as the mathematical formula for these is exactly equivalent. What is interesting is that there seems to be a relation $S_{21} = -\Omega_{21}$.

The physical meaning of the off-diagonal elements strain-rate tensor is an act of shear

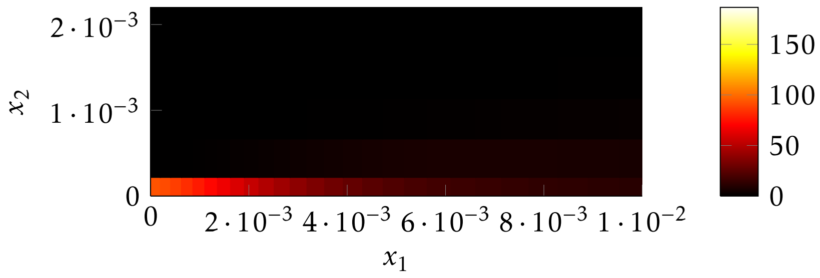


Figure 5: *The dissipation Φ at the lower wall, near the inlet (the dissipation is similar at the upper wall)*

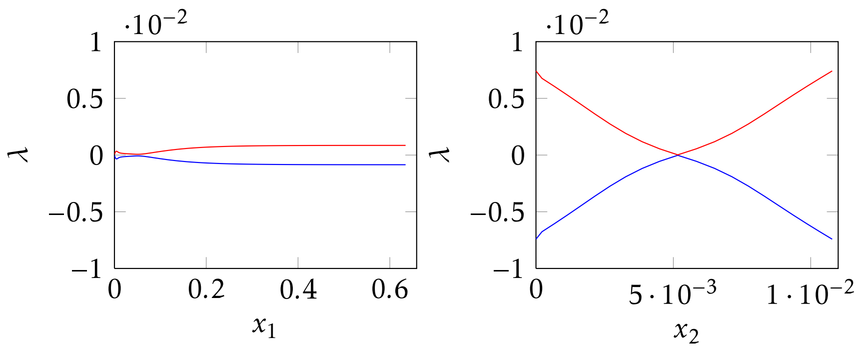


Figure 6: *Eigenvalues w.r.t. x_1 and x_2 at the center of the flow in the developed area.*

on the fluid; in this case, the center moving more quickly than the top and bottom. The physical meaning of the vorticity tensor, however, is one of rotation — in our case, the top half rotates counter-clockwise and the bottom half clockwise. This is confirmed by figure 4.

7 Dissipation

The physical meaning of dissipation is that of energy transfer: from mechanical work (friction) to heat. One can expect the dissipation to be large where shear stresses are high. As figure 5 shows, the dissipation is in fact large close to the walls near the inlet, where figure 1 also showed that shear stress was large. In the fully developed region, dissipation is effectively zero.

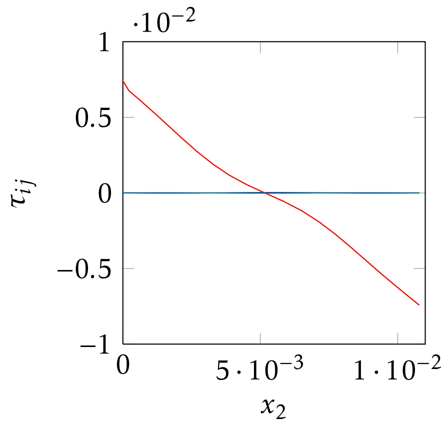


Figure 7: *Stress tensor components w.r.t.* x_2 *in the developed area.*

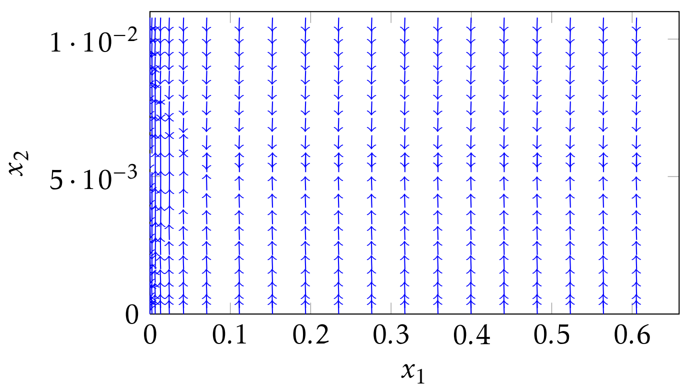


Figure 8: *The eigenvector corresponding to* λ_1 *given at a number of different points in the flow*

8 Eigenvalues

The eigenvalues of τ_{ij}, as illustrated by figure 6, vary with respect to x_1 and x_2. Figure 7 additionally illustrates the four components of τ_{ij} in the developed area, which can be compared to the eigenvalues with respect to x_2 shown in figure 6. Most of them are small and can be neclected, but the one that is large has an interesting relationship with the eigenvalues.

9 Eigenvectors

A quiver plot of the eigenvectors corresponding to the largest eigenvector of τ_{ij} can be seen in figure 8. It basically shows the direction in which the strain âĂIJpullsâĂİ the hardest: towards the middle of the stream.