Task K3

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1 Common task

The common task consists of implementing and solving the k–ε model for a fully developed turbulent channel flow. The channel has a height 2δ and is driven by the constant pressure gradient $\frac{\partial P}{\partial x}$. The flow is also symmetrical, so the computational domain is $0 \leq y \leq \delta$. DNS data is also available for comparison.

Letting $\delta = 1$, we can additionally show that

$$-\int_0^{2\delta} \frac{\partial P}{\partial x}\,\mathrm{d}y = 2\tau_w$$
$$\implies -2\delta \frac{\partial P}{\partial x}\,\mathrm{d}y = 2\tau_w$$
$$\implies -\frac{\partial P}{\partial x} = \tau_w,$$

which if we set $\rho = \delta = u_r = 1$ yields $\frac{\partial P}{\partial x} = 1$.

At $y = \delta$, we set Neumann boundary contitions for U, k and ε. At $y = 0$, we have a Neumann boundary condition for ε and Dirichlet boundary conditions $U = k = 0$.

1.1 Turbulence model

The k–ε equations, as explained by the task, reduce to the 1D diffusion equation encountered in task K1 but with more complicated source terms. Using the discretization given by Ljungskog and Sigurdhsson (2012a) but replacing the source terms and reducing it to one dimension we thus get

$$a_P^\Phi \Phi_P = a_E^\Phi \Phi_E + a_W^\Phi \Phi_W + S_U^\Phi$$

for $\Phi \in \{U,k,\varepsilon\}$. The coefficients $a_P^\Phi, a_E^\Phi, a_W^\Phi$ are interpolated as in Ljungskog and Sigurdhsson (2012a) and Ljungskog and Sigurdhsson (2012b) for all three equations, but the source terms S_P^Φ, S_U^Φ vary:

$$\begin{aligned} S_P^U &= 0, & S_U^U &= \Delta y \\ S_P^k &= -\Delta y \frac{\varepsilon_P}{k_P}, & S_U^k &= \Delta y P_P \\ S_P^\varepsilon &= -\frac{\varepsilon_P}{k_P}\Delta y c_{\varepsilon 2}, & S_U^\varepsilon &= \frac{\varepsilon_P}{k_P}\Delta y c_{\varepsilon 1} P_P \end{aligned}$$

Here, P_P is the value of P_k in the current node, *i.e.*

$$P_P = c_\mu \frac{k_P^2}{\varepsilon_P}\left(\frac{U_N - U_S}{\Delta y}\right)^2.$$

1.2 Implementation

The implementation uses the Gauss-Seidel solver previously used in Ljungskog and Sigurdhsson (2012a) and Ljungskog and Sigurdhsson (2012b), but solving for all three equations instead of just one. The residual is implemented as $\max R_\Phi$, where

$$R_\Phi = \frac{1}{F}\sum_{\text{nodes}} |a_N^\Phi \Phi_N + a_S^\Phi \Phi_S + S_U^\Phi - a_P^\Phi \Phi_P|$$

as usual and $F = U^2 \Delta y$.

1.3 Results

As shown by figure 1 on the following page, the calculated values of U, k and ε are very inexact. Compared to the DNS, all three variables are roughly half as large at some points, and while the k and ε solutions are fairly exact furter out into the channel, the U solution becomes worse.

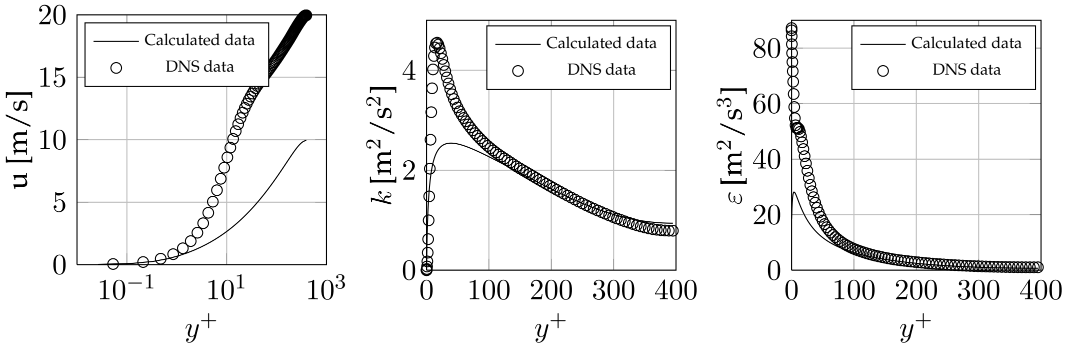
2 Private task

2.1 Turbulence model

The private task consists of using another turbulence model to solve the same problem. The turbulence model used here is the k–ω turbulence model described by Bredberg and Peng (2002). In addition to the usual U equation, we have the k and ω equations as follows:

$$P_k - C_k k\omega + \frac{\partial}{\partial y}\left(\left(\nu + \frac{\nu_t}{\sigma_\omega}\right)\frac{\partial k}{\partial y}\right) = 0$$

$$C_{\omega 1}\frac{\omega}{k}P_k - C_{\omega 1}\omega^2 + \frac{C_\omega}{k}\left(\nu + \nu_t\right)\frac{\partial k}{\partial y}\frac{\partial \omega}{\partial y} + \frac{\partial}{\partial y}\left(\left(\nu + \frac{\nu_t}{\sigma_\omega}\right)\frac{\partial \omega}{\partial y}\right) = 0$$



(a) *Mean velocity U* **(b)** *Turbulent kinetic energy k* **(c)** *Dissipation* ε

Figure 1: *Results of the common task*

where $P_k = ...$ denotes the modelled production term of turbulent kinetic energy. The turbulent viscosity is calculated as $\nu_t = C_\mu f_\mu \frac{k}{\omega}$ where

$$f_\mu = 0.09 + \left(0.91 + \frac{1}{R_t^3}\right)\left(1 - e^{-\left(\frac{R_t}{25}\right)^{2.75}}\right)$$

and $R_t = \frac{k}{\nu\omega}$ is the local turbulent Reynolds number. The constants are

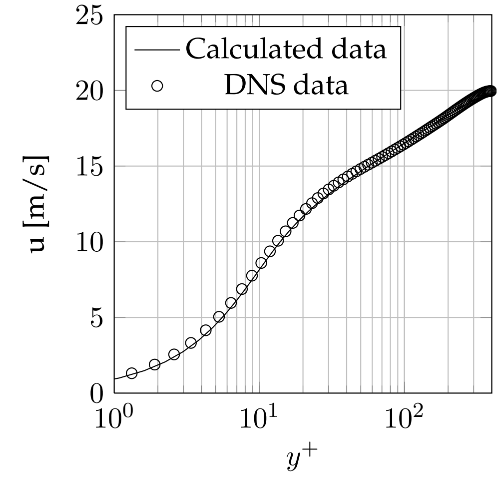
$$\begin{aligned}
&C_k = 0.09 \quad C_\mu = 1, \\
&C_\omega = 1.1, \quad C_{\omega 1} = 0.49, \quad C_{\omega 2} = 0.072 \\
&\sigma_k = 1, \quad \sigma_\omega = 1.8
\end{aligned}$$

These equations are discretized as in the common task, which yields source terms as follows:

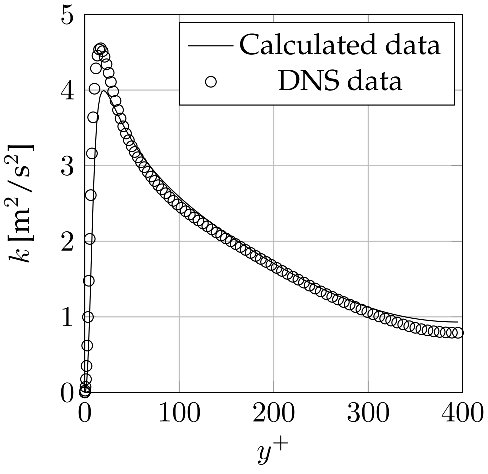
$$\begin{aligned}
S_P^U &= 0, & S_U^U &= \Delta y \\
S_P^k &= -\Delta y \omega_P c_k, & S_U^k &= \Delta y P_P \\
S_P^\omega &= -\omega_P \Delta y c_{\omega 2}, & S_U^\omega &= \frac{\omega_P}{k_P}\Delta y c_{\omega 1} P_P
\end{aligned}$$

Since the cross-diffusion term $\frac{C_\omega}{k}(\nu + \nu_t)\frac{\partial k}{\partial y}\frac{\partial \omega}{\partial y}$ will change its sign in different parts of the channel, we need to put it in S_p if it is negative and in S_u otherwise. This is done in order to ensure that the coefficient matrix is diagonally dominant, which is a sufficient condition for the Gauss-Seidel solver to converge.

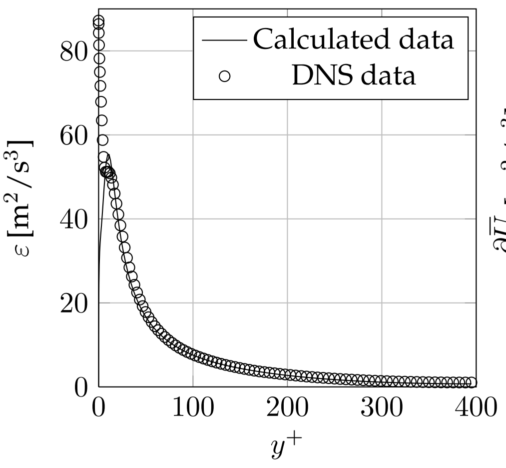
At $y = \delta$, we set Neumann boundary contitions for U, k and ω. At $y = 0$, we have Dirichlet boundary conditions $U = 0$, $k = \infty$ and $\omega = 2\frac{\nu}{c_k(\Delta y)^2}$.



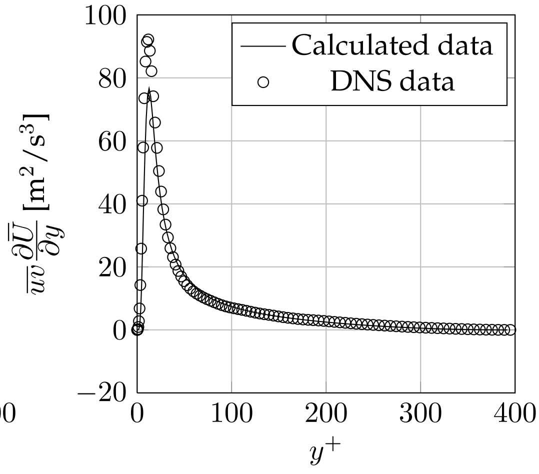
(a) *Mean velocity U*



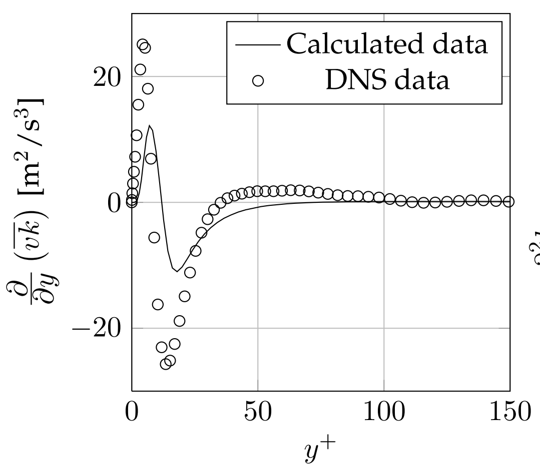
(b) *Turbulent kinetic energy k*



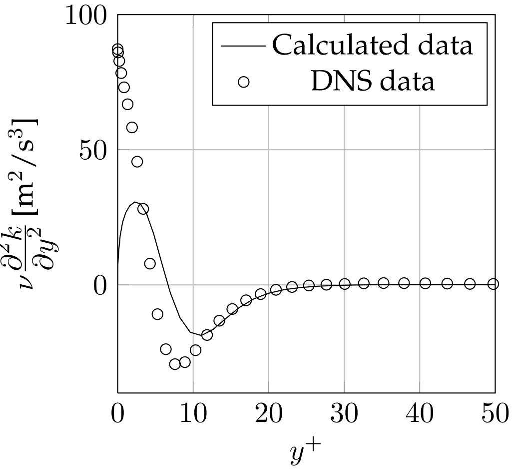
(c) *Dissipation ε*



(d) *Production term*



(e) *Turbulent diffusion*



(f) *Viscous diffusion*

Figure 2: *Results of the private task*

2.2 Implementation

As in the common task, the implementation uses the Gauss-Seidel solver previously used in Ljungskog and Sigurdhsson (2012a) and Ljungskog and Sigurdhsson (2012b). The residuals are identical, bul calculated for ω instead of ε.

2.3 Results

Figure 2 on the previous page shows the solution using the k–ω model compared to DNS data. It is obvious that this model performs better than the model used in the common task, and the solution for U is very close to the DNS data. Only the turbulent and viscous diffusion, shown in figures 2e to 2f on the preceding page, diverge significantly from the DNS data.

The large turbulent length scales shown in figure 4 correspond very well to the DNS data near the wall, but differ greatly near the center of the channel. Figure 3 on the following page shows the Kolmogorov scales — these also correspond well to the DNS data near the wall (but not next to the wall) and differ near the center of the channel. The velocity scale corresponds well to DNS data in the channel center as well.

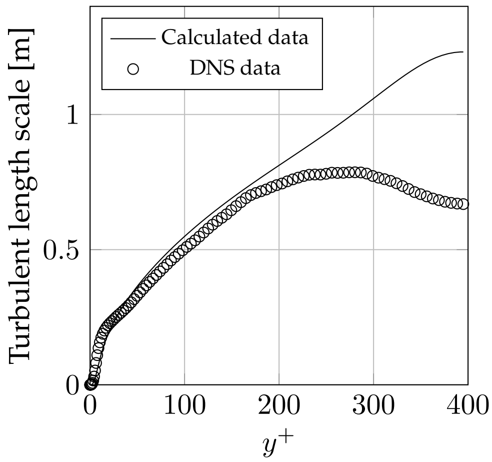


Figure 4: *Large turbulent length scales*

In conclusion, the k–ω model described by Bredberg and Peng (2002) performs much better than the regular k–ε model, at least for this particular flow.

References

Bredberg, J. et al. (2002) An improved k—ω turbulence model applied to recirculating flows. *International Journal of Heat and Fluid Flow*, vol. 23, pp. 731–743.

Ljungskog, E. and Sigurdhsson, S. (2012a) *Task K1*. Unpublished.

Ljungskog, E. and Sigurdhsson, S. (2012b) *Task K2*. Unpublished.

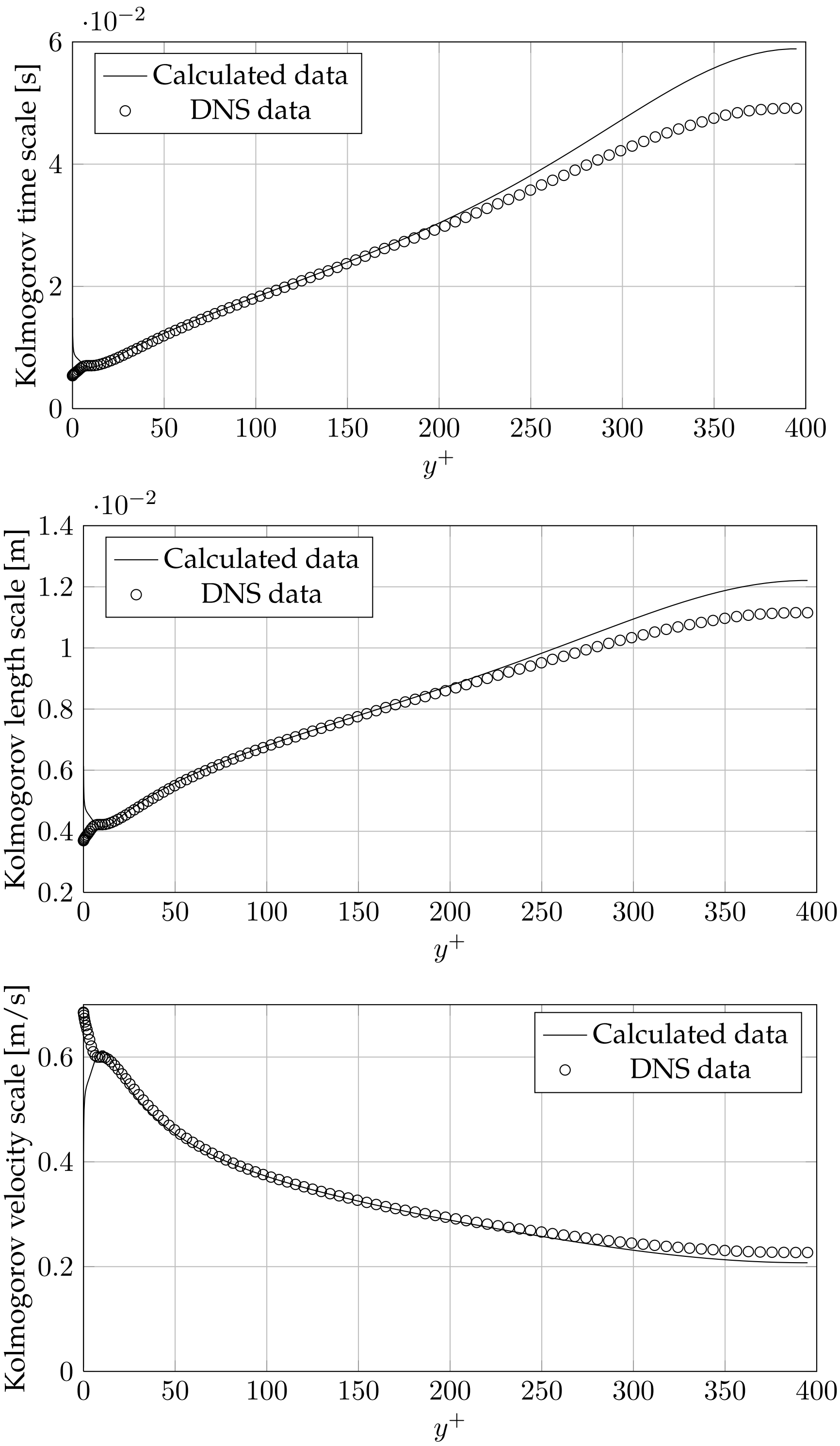


Figure 3: *Kolmogorov scales*