MTF072 – Computational Fluid Dynamics, 14th December 2012

Task K2

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This report discusses the solution of the two-dimensional transport equation,

$$\frac{\partial}{\partial x}(\rho U T) + \frac{\partial}{\partial y}(\rho V T) = \frac{\partial}{\partial x}\left(\Gamma \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\Gamma \frac{\partial T}{\partial y}\right) + S,$$
$$\Gamma = \frac{k}{c_p},$$

discretized as

$$a_P T_P = a_E T_E + a_W T_W + a_N T_N + a_S T_S + b\Delta x \Delta y$$

and solved using the Gauss-Seidel and TDMA iterative methods. The problem is considered on a given grid with velocity fields U and V, $\rho = k = 1$ and $c_p = 500$. The domain is governed by Neumann boundary condition on boundaries 1, 3 and 4 (south, north and west) and a Dirichlet boundary condition with $T = 5\,°\mathrm{C}$ on the eastern wall with the inlet and outlet. The inlet temperature is $T_A = 20\,°\mathrm{C}$.

Implementation

The code, written in MATLAB, consists of a number of functions to implement reading the given mesh and associated velocity data, and iterative solver and visualization of the obtained results. The code starts by reading the mesh, which defines the faces of the control volumes. A node is then placed in the center of each cell. As in general for the finite volume method, all values in a cell is stored in its node which means that the values at the faces have to be interpolated.

The TDMA solver

The TDMA solver is slightly more cumbersome to implement than the Gauss-Seidel described by Ljungskog and Sigurdhsson (2012). The algorithm itself is thoroughly explained by both Versteeg and Malalasekera (2007, pp. 212–215) and Davidson (2005), and the variant implemented alternates between solving in the x and y directions.

Convergence criteria

The convergence criteria, as in Ljungskog and Sigurdhsson (2012), is based on the residual R. In this problem, normalizing the residual using the total heat flux is not optimal. Instead, the heat flux is normalized using the inlet mass flux multiplied by the temperature difference between in- and outflow:

$$F = (\rho U h)_A \Delta T.$$

Results

Standard case

As is evident by figure 1, the difference between the Gauss-Seidel and TDMA solvers in terms of accuracy is small when the solution has converged. The major difference between the two methods is instead running time; TDMA converges after much fewer iterations.

The actual solutions given are shown in figures 3 to 4 on the next page. The flow of the domain is vaguely visible through the temperature; a vortex can be seen in the center. All in all, the results are expected — temperature is high near the inlet and low along the wall, and dissipates along the flow as well as diffusing in the usual manner. Since the TDMA solver is faster, it has been used to calculate all subsequent solutions discussed in this report. Figure 2 also shows the temperature as a function of distance along the northern wall, which has a Neumann boundary condition.

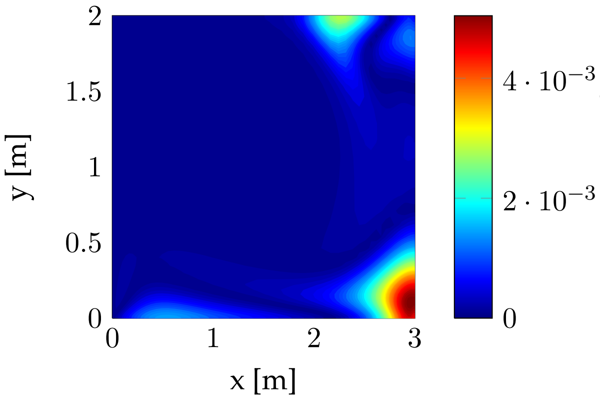

Figure 1: *Difference between Gauss-Seidel and TDMA solvers.*

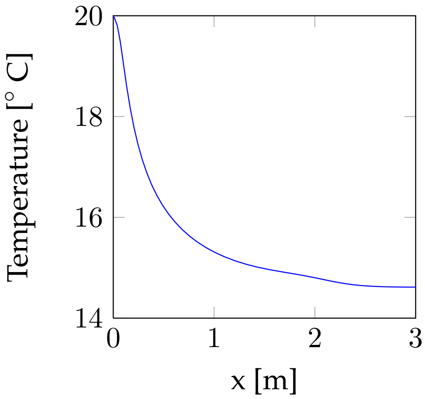

Figure 2: *Temperature at the north boundary.*

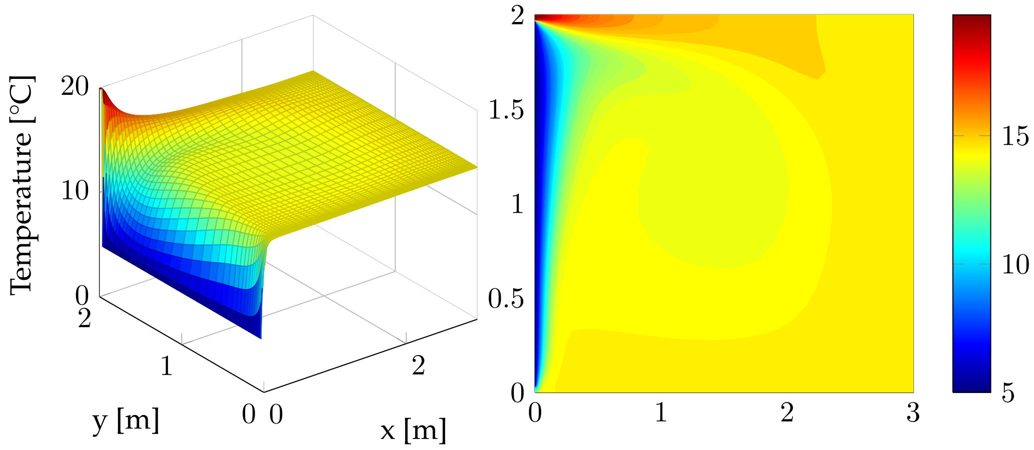

Figure 3: *Temperature field using Gauss-Seidel solver.*

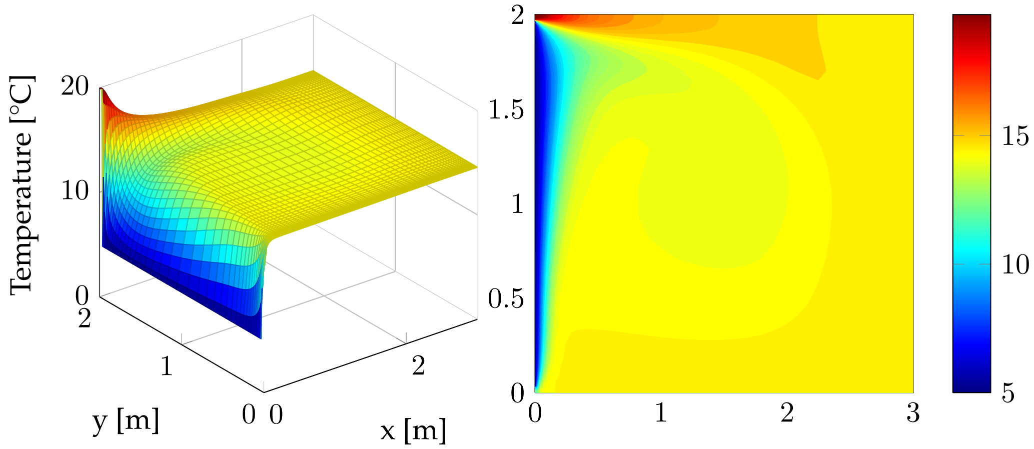

Figure 4: *Temperature field using TDMA solver.*

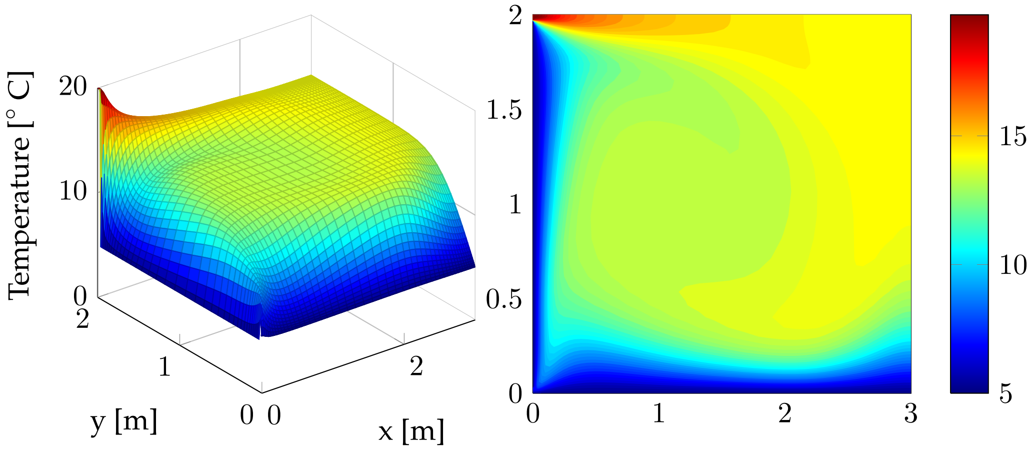

Figure 5: *Temperature field using TDMA solver and Dirichlet boundary condition at south boundary.*

Changing boundary conditions

Substituting the boundary condition on the south wall with a Dirichlet condition (with $T = T_A$) yields the solution shown in figure 5. Comparing this to figure 4 on the preceding page the average temperature is now somewhat lower, and the flow of the liquid reveals itself through the temperature values.

Varying the convergence criteria ε

Varying the convergence criteria ε does not affect the solution to any significant degree. As evidenced by figures 6 to 7 on the next page, increasing ε degrades the solution near the corners, while decreasing it has no visible effect.

Varying heat conductivity k

Figures 8 to 9 on page 6 depict the problem solution for modified k. As expected, a large k (good heat conduction) of the material means that most of the head will leave the domain through the Neumann boundaries, leaving the temperature constant close to T, the temperature on the Dirichlet boundary. conversely, a small k (bad heat conduction) will leave the temperature constant close to the inlet temperature T_A as evidenced by figure 9 on page 6.

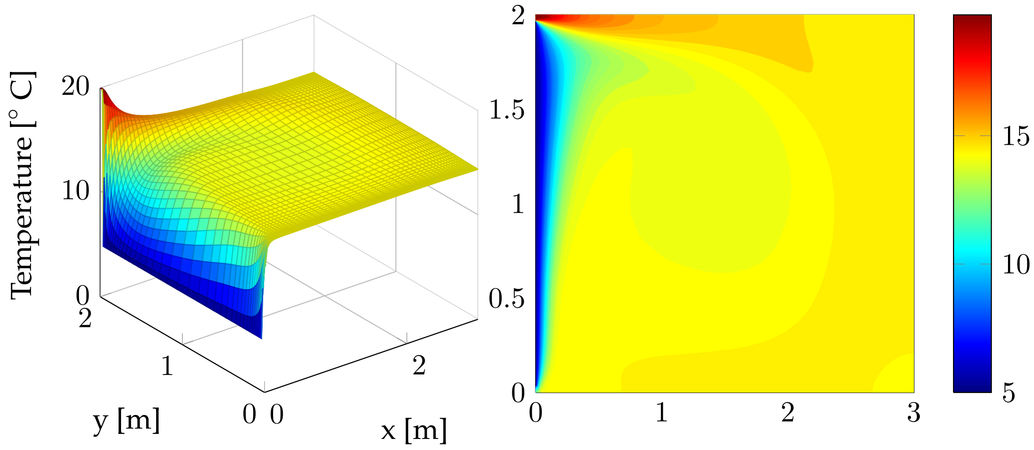

Figure 6: *Temperature field using TDMA solver and* $\varepsilon = 0.01$.

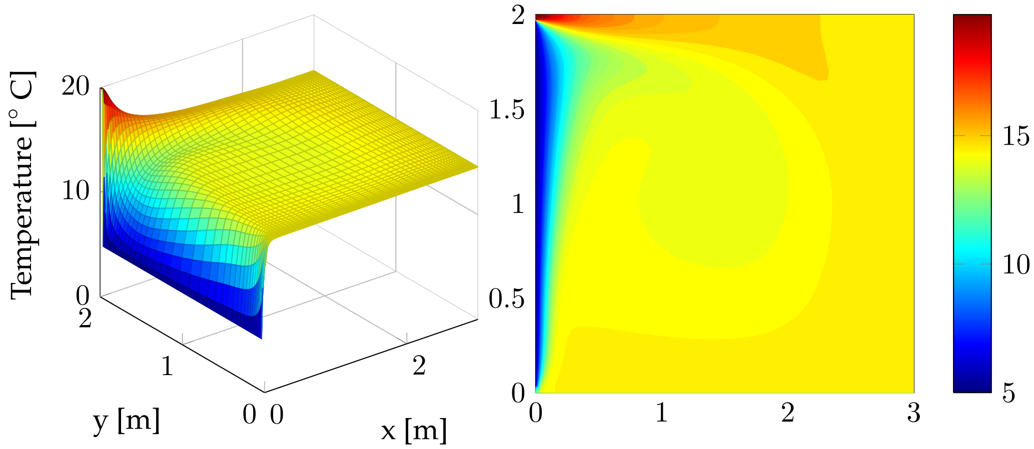

Figure 7: *Temperature field using TDMA solver and* $\varepsilon = 0.0001$.

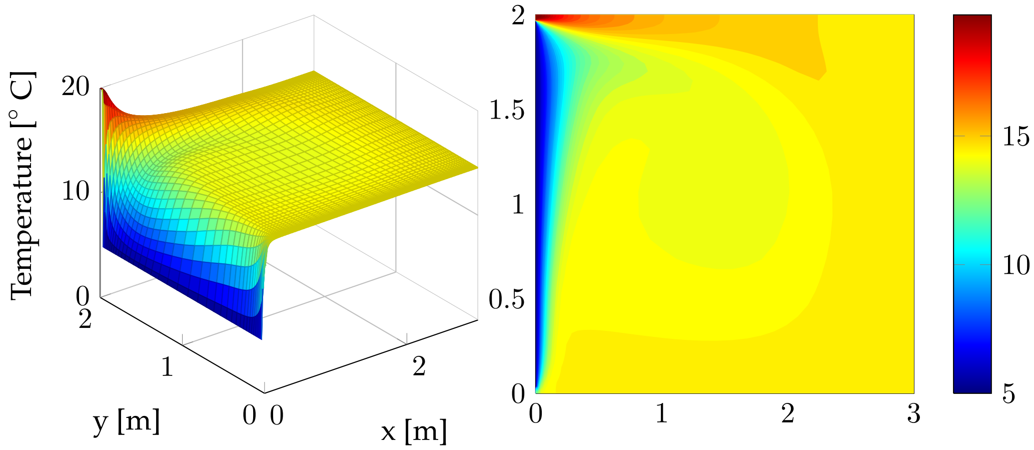

Figure 8: *Temperature field using TDMA solver and* $k = 100$.

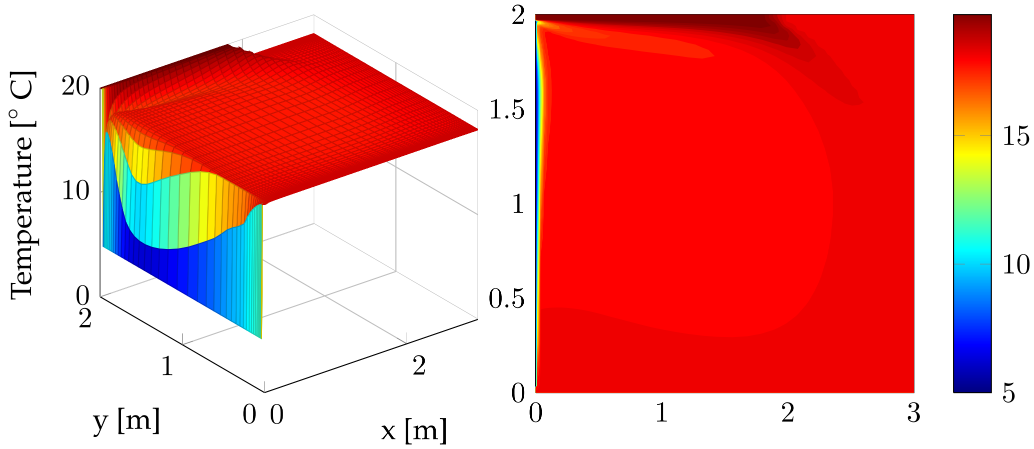

Figure 9: *Temperature field using TDMA solver and* $k = 0.01$.

Heat flux and global conservation

The normalized net flux on the boundaries is $4.8\,\%$ of the net convective flux through the domain. This is fairly close to global conservation, where you'd expect the normalized net flux to be 0.

References

Davidson, L. (2005) Chapter 7: TDMA. In *Numerical Methods for Turbulent Flow*. [Electronic] http://www.tfd.chalmers.se/~sinisa/images/stories/courses/2012_2013/MTF072_CFD/lectureNotes/chapter_7.pdf.

Ljungskog, E. and Sigurdhsson, S. (2012) *Task K1*. Unpublished.

Versteeg, H. and Malalasekera, W. (2007) *An Introduction to Computational Fluid Dynamics: The Finite Volume Method (2nd Edition)*. Upper Saddle River, New Jersey: Prentice Hall.